Technical Comments

Comment on "A Comparison of Solutions to a Blunt Body Problem"

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DR. Van Tuyl of the Naval Ordnance Laboratory (NOL) informs me that the expression "correction to its formulation," paragraph 6[1] is misleading. Only an algebraic error was corrected in his equations to give the improved NOL curve in Fig. 1.

Reference

[1] Perry, J. C. and Pasiuk, L., "A comparison of solutions to a blunt body problem," AIAA J. 4, 1425–1426 (1966).

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Comments on a Proposed Variation of the Cohen-Reshotko Method in Boundary-Layer Theory

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A VARIATION in the method developed by Cohen and Reshotko[1] for calculating laminar boundary-layer properties in a compressible flow with streamwise pressure gradients was recently presented by N. Ness.[2] To the present authors, his results appear to be rather unsatisfactory in that they seem to fail to account for all of the significant parameters. It is the purpose of the present comments to discuss this shortcoming, and to recommend changes that yield a revised method that should be more universally acceptable.

The Cohen-Reshotko method requires knowledge of the distribution along the body of the so-called correlation number, $n(x)$. This is a parameter which is proportional to the square of the local momentum thickness, and this fact was used by Ness in outlining a new method for its determination. He used the Karman momentum integral equation, together with the concept of local similarity, to develop the following expression [Eq. (14) of Ref. 2] for the local momentum thickness (squared)

$$\theta(x)^2 = 2(1+\epsilon)\,\theta^2_{\text{stag}}\left(\frac{du_e}{dx}\right)_{\text{stag}} \times \left(1+\frac{\gamma-1}{2}M_e^2\right)^{(2-\gamma)/(\gamma-1)} \frac{\int_0^x p_e^{\alpha}u_e r_w^{2\epsilon}dx}{p_e^{\alpha}u_e^2 r_w^{2\epsilon}} \tag{1}$$

Since the preceding result was partially based on the assumption of local similarity, it is natural to compare it to what is obtained from local similarity exclusively. This result is given below:

$$\theta(x)^2 = 2(1+\epsilon)\,\theta^2_{\text{stag}}\left(\frac{du_e}{dx}\right)_{\text{stag}} \times \left(1+\frac{\gamma-1}{2}M_e^2\right)^{(2-\gamma)/(\gamma-1)} \frac{\theta_{\text{tr}}(x)^2}{\theta_{\text{tr}}(0)^2} \frac{\int_0^x p_e^{\alpha}u_e r_w^{2\epsilon}dx}{p_e^{\alpha}u_e^2 r_w^{2\epsilon}} \tag{2}$$

In the preceding expression, the quantity $\theta_{\text{tr}}(x)$ is the transformed momentum thickness defined as

$$\theta_{\text{tr}}(x) = \int_0^{\infty} f_\eta(1-f_\eta)d\eta \tag{3}$$

In the local similarity method, θ_{tr} depends on the local values of three variables: 1) the pressure gradient parameter β, defined by Eq. (4) of Ref. 2; 2) the wall-to-freestream stagnation temperature ratio, $T_w/T_{e_{\text{stag}}}$, and 3) a dissipation parameter, which is proportional to the square of the local Mach number M_e, if the Prandtl number is not one. The quantity θ_{tr} is evaluated from similar solutions to the momentum and energy equations, such as those presented in Refs. 3 and 4. Clearly, Eq. (2) differs from Ness' results by the appearance of the ratio $[\theta_{\text{tr}}(x)/\theta_{\text{tr}}(0)]^2$. This ratio can be interpreted as a correction factor which partially accounts for the effects of the variation of β, $T_w/T_{e_{\text{stag}}}$, and M_e along the body.

The fact that this correction factor is not present in Eq. (1) is what appears to be unsatisfactory, as can be illustrated by the following example. Consider a blunted wedge in supersonic flow where the ratio of nose radius to base height is very much less than one. It is expected that far back from the nose, the boundary-layer development is essentially similar to that on a flat plate. Calculations of the momentum thickness by Eq. (2) would bear this out, and in fact, in the limit of zero nose radius, this equation yields the flat plate result exactly. On the other hand, the momentum thickness calculated by Eq. (1) would differ by the factor $\theta_{\text{tr}}(x)/\theta_{\text{tr}}(0)$, which in this example, has a limiting value of about 1.6 when the wall is adiabatic, and 0.9 when the wall is very cold. These factors are squared in the correlation number, which is used to determine other physical characteristics such as heat transfer, and are thus quite significant. This example gives a rather clear indication that the parameter θ_{tr} is important. The next question of concern is why it does not appear in Eq. (1).

To answer this, the derivation outlined by Ness has been reviewed in detail and the following perhaps subtle point is noted. Ness states (correctly) that in the approximation of local similarity, the group of terms he calls F in the Karman integral equation becomes

$$F_{\text{loc sim}} = C\,\frac{T_e^2}{T_{\text{ref}}^2}\,\frac{p_{\text{ref}}^{\alpha}}{p_e^{\alpha}}\,\theta_{\text{tr}}^2 \tag{4}$$

He then "seeks an expression for F which, when approximated by local similarity, yields the aforementioned $F_{\text{loc sim}}$." This he chooses to write as

$$F = \frac{1}{2}\,\frac{1}{\xi}\,\frac{d\xi}{dx}\,\frac{u_e\theta^2}{\nu_{\text{ref}}} \tag{5}$$

where

$$\xi = \frac{C\rho_{\text{ref}}\,\mu_{\text{ref}}}{p_{\text{ref}}}\int_0^x p_e^{\alpha}u_e r_w^{2\epsilon}dx$$

and substitutes it into the Karman integral equation which,

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